The Operation of Bolometers Under Pulsed Power Conditions*

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Summary—**The dynamic behavior of Wollaston (thin platinum) wire bolometers under pulsed power conditions is examined and shown to be responsible for certain errors in the measurement of microwave power. These errors, in order of importance, arise from the nonlinearity of bridge null arm current as a function of bolometer resistance, from the nonlinear cooling of the bolometer in the interval between pulses, from the time variation of VSWR of the bolometer and from the time variation of bias power in the bolometer. The total error is found to be proportional to the bolometer resistance excursion during a pulse; the resistance excursion, in turn, is proportional to the pulse energy. All of these errors, with the exception of that due to nonlinear cooling, are amenable to calculation. The latter error was evaluated by subtracting the sum of the computed errors from the experimentally determined total error. The total error is about 14 to 15 per cent per 100 ohms resistance excursion for two widely used bolometer types under typical conditions. More than half of the above error may be eliminated by suitable design of the bridge circuitry.**

INTRODUCTION

WHEN THERMALLY fast elements such as Wollaston (thin platinum) wire bolometers are used to measure pulsed rf power certain errors arise from the dynamic behavior of the bolometer which are entirely absent in cw measurements.

A discussion of one type of error for the case of modulated power was given in 1947,[1] but in that analysis it was assumed, in effect, that the bolometer resistance completely follows the modulation envelope. In this paper the finite time-constant of the bolometer is explicitly taken into account and an evaluation is made of the effect of sources of error which have not been treated in any detail elsewhere. The exchange of heat between the bolometer and its surroundings is a rather complex thermal problem. In an equivalent circuit representation the heat capacity of the wire is shunted by the thermal analog of an RC transmission line representing the surrounding media (air, bolometer supports, etc.) to which some of the heat in the wire is transferred. Thus, the input response of this system must be expressed in terms of a multiplicity of time-constants corresponding to the natural modes of the equivalent network. The problem is further complicated by the nonlinear nature of the convective cooling of the wire. Fortunately the precise value of time-constant is not critical for the evaluation of errors in measuring pulsed power and in this paper a single representative time-constant is used.

The usual procedure for measuring rf power, particularly at microwave frequencies, is to substitute low-frequency power. A method commonly used is to bias the bolometer with dc power in a Wheatstone bridge to the point where the bridge is balanced. The incident rf power is measured by retracting bias power until the bridge is again balanced. The rf power is then equated to the bias power that has been retracted. The above procedure may be subject to an inherent error—the so-called "substitution" error—but this occurs in the case of both modulated and cw power and has been treated in detail elsewhere.[2,3]

When the bolometer is subjected to pulsed rf power its temperature and resistance may undergo rather large periodic time variations as a function of the modulation envelope, with the following effects on the measurement:

1. The time-average of the bolometer resistance will be different at initial and final balance of the bridge; consequently the retracted bias power will not exactly equal the applied rf power and a measurement error will be incurred. This error is due to the nonlinear dependence of galvanometer current on bolometer resistance and is referred to as "bridge nonlinearity" error. The reason for this is made clear when the expression for the galvanometer current in a Wheatstone bridge circuit is examined. The galvanometer current, i_g, may be expressed in general as

$$i_g = k\frac{R_1 - R_b(t)}{r + R_b(t)}, \tag{1}$$

where $R_b(t)$ is the time-dependent bolometer resistance and k, r and R_1 are constants determined by the bridge parameters. At initial balance of the bridge, with only bias power applied, the bolometer resistance is, of course, equal to R_1. When pulsed rf power is applied, the bolometer resistance varies with time and causes a similar but not proportional fluctuation in galvanometer current. The requirement for bridge balance in the latter case is that the time-average galvanometer current $\bar{i}_g$ be zero, or

$$\bar{i}_g = \frac{1}{T}\int_0^T i_g dt = 0 \quad (T = \text{pulse repetition period}) \tag{2}$$

The above condition does not necessarily imply that the time-average resistance of the bolometer is the same value, R_1, which was obtained at initial balance. Only if i_g is a linear function of $R_b(t)$ will this be true, and the extent of the error will be smaller as the time varying portion of $R_b(t)$ is smaller in relation to the constant

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[1] T. Moreno and O. C. Lundstrom, "Microwave power measurement," PROC. IRE, vol. 35, pp. 514-518; May, 1947.

[2] H. J. Carlin and M. Sucher, "Accuracy of bolometric power measurements," PROC. IRE, vol. 40, pp. 1042–1048; September, 1952.

[3] E. Weber, "On microwave power measurements," *Elektrotechnik Und Maschinenbau*, 71st year, pp. 254–259; September, 1954.

portion of the denominator of i_g; that is, the more linear the relation between i_g and $R_b(t)$.

2. A given amount of pulsed rf power will in general produce a smaller time-average resistance change in a bolometer than will the same power applied in cw fashion. This is due to the convective cooling of the bolometer which is a nonlinear function of the temperature rise of the bolometer. The convective heat loss per unit length of wire is proportional to $\theta^{1.2}$ where θ is the temperature rise above ambient. As a result, in the interval between pulses, when no rf power is being applied and the bolometer is cooling, its effective thermal time constant will be shorter the higher the temperature to which it has been heated by a pulse of power. The average temperature and resistance of the bolometer will be correspondingly less than the value obtained under cw conditions. This error is referred to as "bolometer nonlinearity" error.[4]

3. The rf impedance of the bolometer will vary during the pulse so that, on the average, some power will be reflected despite the fact that the average bolometer resistance may be such as to give a perfect match to the transmission line. This type of error may be enhanced by generator mismatch.

4. The bias power in the bolometer will fluctuate as the bolometer resistance varies with time. The retracted bias power will not be correctly calculated unless the true time-average of the fluctuating bias power is used.

Errors 2 and 3 are inherent in the bolometer while errors 1 and 4 are inherent in the external instrumentation. All of these errors result in a power reading that is lower than the true value.[5] From a knowledge of the thermal and electrical characteristics of the bolometer it is possible to obtain analytically the bolometer response to a given set of pulsed power conditions and to calculate all but the bolometer nonlinearity error. The latter is best determined experimentally by carefully correcting for the other errors.

Bolometer Response to Pulsed Power

To calculate the measurement error under a given set of operating conditions the response of the bolometer to a time varying power input must be known. This can be found from a solution of the heat balance equation of the bolometer and from the known relationship between the resistance of the bolometer and its temperature. A simple approximation[6] to the heat balance equation which is adequate for the purpose is

$$c_p \frac{d\theta}{dt} + \gamma\theta = P(t), \tag{3}$$

where θ is the temperature rise of the bolometer above ambient, c_p its heat capacity, γ the coefficient of heat loss to the surroundings, $P(t)$ the time-dependent power input into the bolometer and t the time. The equation states that the time rate of energy dissipation in the bolometer is equal to the sum of $\gamma\theta$, the time rate of energy loss to the surroundings, and $c_p d\theta/dt$, the time rate of energy storage as heat in the bolometer.

The bolometer time-dependent resistance $R_b(t)$ is related to $\theta(t)$ by

$$R_b(t) = R_0[1 + \alpha\theta(t)], \tag{4}$$

where R_0 is the resistance of the bolometer at ambient temperature and α is the temperature coefficient of resistivity of the metal (platinum) evaluated at ambient temperature.

Let us first consider the response of the bolometer to a suddenly applied constant power P_0. The temperature of the bolometer will rise exponentially to its steady-state value θ_0 with a characteristic time constant, τ, according to the equation

$$\theta = \theta_0(1 - \epsilon^{-t/\tau}), \tag{5}$$

where $\theta_0 \equiv P_0/\gamma$ and $\tau \equiv c_p/\gamma$. The corresponding steady-state resistance change of the bolometer, according to (4), will be

$$R_b - R_0 = \alpha R_0 \theta_0 = (\alpha R_0/\gamma)P_0 = sP_0. \tag{6}$$

The resistance change is therefore proportional to the power input P_0; the proportionality constant $s \equiv \alpha R_0/\gamma$ expresses the resistance change per unit power input and is denoted as the bolometer power sensitivity factor.

If the bolometer is subjected to a train of rectangular pulses of peak power level P, pulse width δ and repetition period T, its steady-state response will be a rise of resistance during a pulse and a decay of resistance between successive pulses, as given by (7) and (8) below. For the duration of the pulse $(0 \leqq t \leqq \delta)$,

$$R_b(t) = R_0 + sP\left[1 - \epsilon^{-t/\tau}\frac{1 - \epsilon^{-(T-\delta)/\tau}}{1 - \epsilon^{-T/\tau}}\right]. \tag{7}$$

In the interval between pulses $(\delta \leqq t \leqq T)$

$$R_b(t) = R_0 + sP\frac{1 - \epsilon^{-\delta/\tau}}{1 - \epsilon^{-T/\tau}} \cdot \epsilon^{-(t-\delta)/\tau}. \tag{8}$$

When the pulse width δ is much less than the bolometer time-constant τ and the latter much less than the pulse repetition period T, (7) and (8) may be simplified to

$$R_b(t) = R_0 + (\Delta R/\delta)t \qquad \text{when} \quad 0 \leqq t \leqq \delta, \tag{9}$$

$$R_b(t) = R_0 + (\Delta R)\epsilon^{-(t-\delta)/\tau} \quad \text{when} \quad \delta \leqq t \leqq T, \tag{10}$$

where ΔR denotes the resistance excursion of the

[4] The nonlinear convective cooling is also responsible for the nonlinearity in the steady-state resistance-power characteristics of bolometers. However, "static" nonlinearity will not necessarily introduce an error in the measurement of cw power if the substitution procedure is used in an appropriate manner, while "dynamic" nonlinearity introduces an unavoidable error in pulsed power measurements no matter what procedure is used.

[5] This holds for positive temperature coefficient elements such as Wollaston wires. The direction of the bridge nonlinearity error in the case of negative temperature coefficient elements is to make the reading high, but all other errors tend to make the reading low.

[6] The approximation consists in assuming that the wire is uniform in temperature throughout its length, that c_p and γ are independent of temperature and that the heat loss to the surroundings is proportional to the first power of the temperature rise, although it more closely follows a 1.2 power law.

bolometer during the pulse. Note that (9) describes a linear rise in resistance during the pulse and (10) an exponential decay in the interval between pulses. The resistance excursion ΔR may be written in approximate form as follows (for $\delta \ll \tau$):

$$\Delta R \simeq sP\delta/\tau = (\alpha R_0/c_p)P\delta = (\alpha R_0/c_p) \times \text{pulse energy}, \quad (11)$$

since $P\delta$ is the energy in a single pulse of width δ and peak power P.

Eq. (11) shows that the resistance excursion for a narrow pulse is very nearly proportional to the energy of the pulse, the proportionality factor being $\alpha R_0/c_p$. This factor, which strongly governs the magnitude of the total error, depends on the physical dimensions of the wire, being inversely proportional to the fourth power of the radius and independent of the length.[7] Thus, the thinner the wire the larger the resistance excursion per unit pulse energy input. This factor may also be expressed as s/τ, the ratio of the steady-state power sensitivity, $s = \alpha R_0/\gamma$, and the bolometer time-constant, $\tau = c_p/\gamma$.

Pulsed Power Measurement Errors

A study was made of the total error in measuring pulsed rf power using three different bolometer types: a Sperry 821 barretter, a PRD-type 630A mica-mounted bolometer and a "desensitized" PRD bolometer.[8] The rf power was supplied by a 3,000-mc magnetron in one microsecond pulses at a rate of 1,000 pulses per second, and was measured with a dc circuit, the basic form of which is shown in Fig. 1. In the actual measurements, a 200-ohm equal-arm Wheatstone bridge was used in

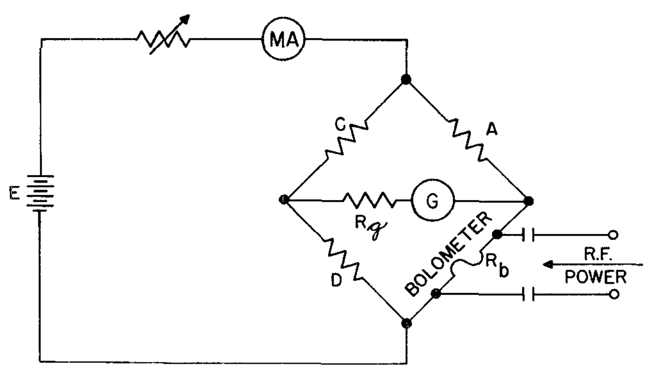

Fig. 1—Basic bolometer bridge circuit.

series with a 24-volt storage battery and decade resistance box. The null detector was a galvanometer of 10 ohms resistance. The error was determined by comparison with a thermistor element which, because it underwent very small resistance variations[9] in response to the pulsed rf power, measured the latter with the same accuracy as it did cw power. Any measurement discrepancy between bolometer and thermistor under cw conditions was applied as a correction to the pulsed power measurements. (Thus, it was found that the thermistor cw power reading was lower than the barretter cw power reading by 3 per cent, a difference which was attributed to thermistor mount inefficiency, since mismatch errors were negligible. The thermistor pulsed power readings were therefore corrected upward by 3 per cent.)

The total error as a function of resistance excursion for the different bolometers is plotted in Fig. 2, and is seen to be proportional to the resistance excursion with

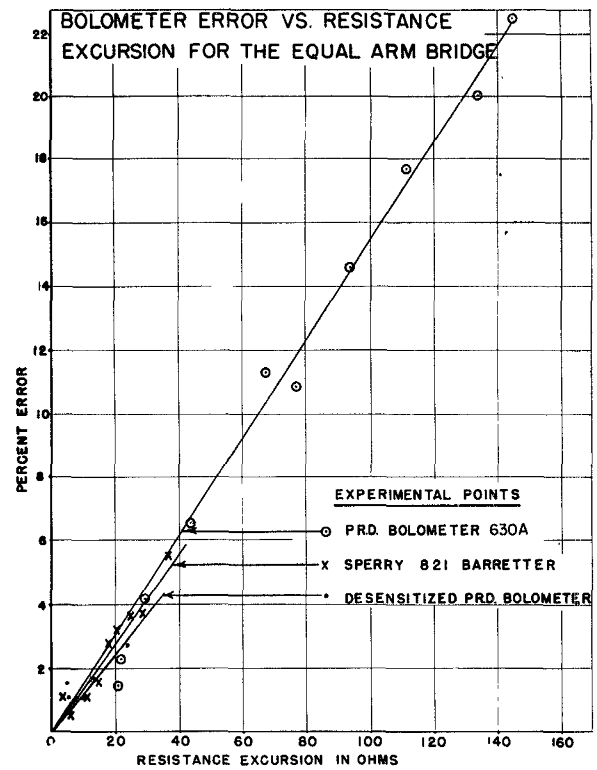

Fig. 2—Experimentally measured error as a function of bolometer resistance excursion during an rf pulse. The measuring circuit consisted of a 200-ohm bridge in series with a 24-volt battery and variable series resistance to provide the necessary bias. The galvanometer resistance was 10 ohms. The rf power was applied in microsecond pulses at 1,000 pps.

a proportionality factor which is approximately the same for all three bolometer types. In Fig. 3 the resistance excursion is shown to be proportional to the pulse energy, each bolometer having its own proportionality factor. Consequently the proportionality of *error* to *pulse energy* is different for each bolometer type, the bolometer with the smallest heat capacity being subject to the largest error on a pulse-energy basis.

The extent to which the various sources of error con-

[7] The factor $\alpha R_0/c_p$ may be written entirely in terms of the radius of the wire and certain physical constants of the metal (platinum). Thus, if σ is the electrical conductivity, ρ_d the density, c_h the specific heat and a the radius, then $\alpha R_0/c_p = \alpha/\pi^2\sigma\rho_d c_h a^4$.

[8] This was a PRD-type 630A bolometer coated with silicone varnish to increase its effective heat capacity. This bolometer had a lower sensitivity than the 630A, and a modified thermal time constant.

[9] This is due to the smoothing effect of the larger heat capacity and longer time-constant of the thermistor on the temperature variations produced by the rf pulses.

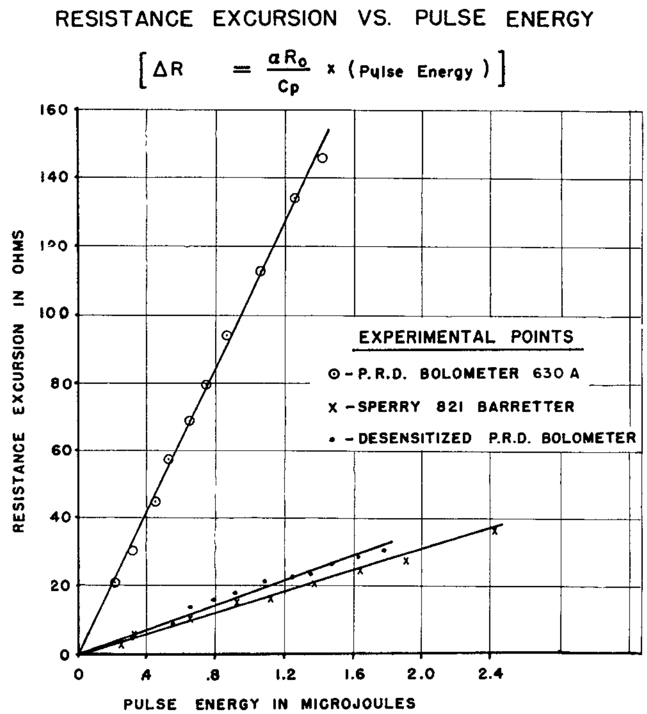

Fig. 3—Bolometer resistance excursion as a function of pulse energy. The resistance excursion is proportional to the pulse energy; the proportionality factor is $\alpha R_0/c_p$; α is the temperature coefficient of resistivity of platinum, R_0 the cold resistance of the bolometer, and c_p its heat capacity.

tribute to the total can be determined by calculation (except for the bolometer nonlinearity error). Each type of error is strongly dependent on the bolometer resistance excursion but, in addition, such factors as the bridge parameters, bolometer time-constant and pulse repetition period are significant. For the type of measuring circuit used in the study of pulsed power errors (equal-arm 200-ohm bridge operated from an essentially constant current source with a low impedance null detector) the bridge nonlinearity error constitutes more than half the total error, while the rf impedance variation and bias power variation errors each contribute about 10 per cent of the total.

A relatively simply approximate expression for the bridge nonlinearity error has been derived which is good to about 5 to 10 per cent and involves a parameter $q=\Delta R/(r+R_0')$, the pulse repetition period T and bolometer time constant τ (see Appendix I). The constant r is the same one used in (1), R_0' is the bolometer resistance immediately before it absorbs the energy of a successive pulse and ΔR is the resistance excursion. For reasonable rf (average) power levels $R_0' \cong R_1$, where R_1 is the bolometer resistance required to balance the bridge. The expression for the error is

$$\text{Per cent error} = 100q\cdot\frac{T\left(\frac{1}{2}-\frac{q}{3}\right)-\tau}{T-q\tau}. \quad (12)$$

For an equal arm bridge of resistance R fed by a constant current source the value of r ranges between R (for a zero impedance null detector) to $3R$ (for an infinite impedance null detector). Since the resistance excursion rarely exceeds R (in most cases being much less than R) the value of q in an equal-arm bridge has a practical upper limit of 0.5. It is seen that the error is very nearly proportional to q and therefore to ΔR. Also if r is made larger, q becomes smaller, the bridge becomes more linear [see (1)], and the error is reduced. Of course, an increase in r makes the bridge less sensitive but this can be compensated for by using a voltage-actuated detector such as a chopper-amplifier. In fact, it is possible by suitable design of the measuring circuitry to eliminate both bridge nonlinearity and bias power variation errors which together make up about two thirds of the total error. This is discussed in the next section.

Table I summarizes the experimental data obtained on the three different bolometers under the operating conditions and with the bridge circuitry described in the first paragraph of this section. The method for calculating the bridge nonlinearity error is derived in Appendix I, the bias power and rf impedance variation errors are treated in Appendix II, while Figs. 4 and 5 give computed data for the last two errors as a function of ΔR.

TABLE I

SUMMARY OF PULSED POWER ERRORS

(pulse width 1 μsec, repetition rate 1,000 pps)

Bolometer type	Resistance excursion (ohms per microjoule)	Per cent error (per 10 ohms excursion)	Approximate resistance excursion at burnout (ohms)	Approximate pulse burnout energy (microjoules)	Approximate error at burnout point per cent
PRD 630A	105	1.54	150	1.4	22
"Densitized" PRD 630A	18	1.25	30	1.8	4.0
Sperry 821	15	1.40	40	2.6	5.5

The sum of the errors due to bridge nonlinearity, bias power variation and bolometer rf impedance variation, as read from computed curves, is tabulated below for

TABLE II

COMPARISON OF COMPUTED AND MEASURED ERRORS

(pulse width 1 μsec, repetition rate 1,000 pps)

Bolometer type	Sum of computed errors	Experimentally measured error	Bolometer nonlinearity error
	per cent	per cent	per cent
PRD 630A (ΔR=100 ohms; temperature swing approx. 175 degrees C.)	10.8	15.6	4.8
Sperry 821 (ΔR=40 ohms; temperature swing approx. 100 degrees C.)	4.1	5.5	1.4

comparison with the total experimental error. The residual error, obtained by subtracting the computed errors from the experimentally measured errors, is shown in Table II as bolometer nonlinearity error.

Reduction of Errors Due to External Circuitry

Examination of the data shows that for the Sperry 821 barretter and "desensitized" PRD-type 630A bolometer the pulsed power measurement errors are moderate even at pulse energies which extend to the burnout limit of these bolometers while in the case of the thermally much lighter PRD-type 630A bolometer the error may be substantial. However, since about 60 per cent of this error is due to the external circuitry and is not inherent in the bolometer itself, the accuracy can be significantly improved by redesign of the bridge circuit.

One method of accomplishing this is to provide the proper asymmetry in the bridge and to use both a constant current supply and a high impedance null detector. To illustrate this, consider the diagram of Fig. 1. The galvanometer current, i_g, is

$$i_g = \frac{I(AD - CR_b)}{(A + C)(R_b + D) + R_g(A + C + D + R_b)}, \quad (13)$$

where I is the current entering the bridge. The detector voltage e_g is given by

$$e_g = i_g R_g = \frac{I(AD - CR_b)}{(A+C)(R_b+D)/R_g + A + C + D + R_b} \cdot \quad (14)$$

For an infinite impedance detector ($R_g = \infty$)

$$e_g = \frac{IC(AD/C - R_b)}{(A + C + D) + R_b} \cdot \quad (15)$$

A comparison of (15) with (1) shows that $(A+C+D)$ may be identified with r and AD/C with R_1 of (1). To linearize the bridge $(A+C+D)$ must be made large relative to R_b. Thus, if C and D are made equal and large relative to A (say $C=10A$), then the parameter q of (12) will be reduced by an appreciable factor (more than five) as compared with the value for an equal arm bridge using a low impedance detector and the bridge nonlinearity error will be correspondingly reduced. At the same time, if C and D are large relative to A most of the current entering the bridge is diverted through the bolometer and the error due to the bias power variation is virtually eliminated. The reason for this is seen when the limiting case is considered in which all of the current I passes through the bolometer. Then the instantaneous bias power is proportional to $R_b(t)$, since I is constant. The time average bias power is equal to $I^2\overline{R}$ where $\overline{R}$ is the time average bolometer resistance. In the absence of any bridge nonlinearity error, $\overline{R}$ is the same at both initial and final balance of the bridge (namely $\overline{R} = R_1 = A$) and the average bias power at final balance is correctly given by I^2R_1 regardless of bias power fluctuations.

Method for Determining Resistance Excursion Factor

It has been pointed out that the most significant variable to which the pulsed power measurement error can be related in the case of short pulses of power is the resistance excursion of the bolometer. The latter is related to the pulse energy by a proportionality factor $\alpha R_0/c_p$ which depends on the radial dimensions of the wire and on the physical properties of the metal. A method has been developed for determining this factor directly by measuring the phase shift in the audio voltage across the bolometer when it constitutes one arm of an ac balanced Wheatstone bridge and a combination of audio and dc power in the bolometer is varied. The experimental arrangement is shown in Fig. 6. By plotting the value of the shunting capacitance, C, required across the adjacent arm of the bridge as a function of the dc power at balance, a straight line is obtained. The slope of this line yields the desired factor $\alpha R_0/c_p$. If the bridge arm resistances are each equal to R, the dc power in the bolometer is P_{dc} and the angular frequency of the audio voltage is ω then, by equation (50) of Appendix III

$$\alpha R_0/c_p = s/\tau = \frac{4\overline{R}^2\omega^2}{7} \cdot \frac{dC}{dP_{dc}} \cdot \quad (16)$$

Conclusion

The rf measurement error of Wollaston wire bolometers under typical pulsed power conditions is essentially proportional to the resistance excursion of the bolometer. For short rf pulses the resistance excursion is proportional to the pulse energy and inversely proportional to the fourth power of the wire diameter. The factor of proportionality may be determined experimentally from the phase shift in audio voltage across the bolometer when connected in an ac balanced Wheatstone bridge to which a combination of audio and dc voltage is applied. The major portion of the error is due to the measurement circuitry. When the latter is modified to include a constant current source, a high impedance null detector, and sufficiently high resistance in the appropriate arms of the bridge, the external instrumentation error may be greatly reduced. In any case this paper shows how to make the necessary corrections for external instrumentation errors when the circuit constants are known. A substantial correction for the remaining inherent bolometer error can be made by use of the data presented in this paper if the bolometer resistance excursion is known. The latter may be found from the pulse energy as determined from the (uncorrected) average power and pulse repetition rate.

Appendix I

Bridge Nonlinearity Error

For any Wheatstone bridge, the galvanometer current may be written as

$$i_g = k\frac{R_1 - R_b(t)}{r + R_b(t)}, \tag{17}$$

where the constants k and r depend on the fixed bridge arm and galvanometer resistances as well as current limiting resistance external to the bridge. R_1 is the value of bolometer resistance when the bridge is initially balanced with bias power only.

By (9) and (10), the bolometer resistance variation during a pulse is

$$R_b(t) = R_0' + (\Delta R/\delta)t, \tag{18}$$

and between successive pulses

$$R_b(t) = R_0' + \Delta R\epsilon^{-(t-\delta)/\tau} \quad \text{when} \quad \delta \leqq t \leqq T. \tag{19}$$

ΔR is the bolometer resistance excursion, δ the pulse width, τ the bolometer time constant and R_0' the resistance the bolometer would have if only the dc bias power at final bridge balance were applied to it.

Substituting (18) and (19) into (17), integrating over a complete period T and setting the integral equal to zero, as would be required for a zero reading on the galvanometer, the following relation is obtained:

$$R_1 - R_0' = \frac{\Delta R}{q}\cdot\frac{(q\tau - \delta)\ln(1+q) + q\delta}{(\delta - q\tau)\ln(1+q) + q(T-\delta)}, \tag{20}$$

where

$$q = \frac{\Delta R}{r + R_0'}; \tag{21}$$

the average bolometer resistance is

$$\overline{R} = R_0' + s\overline{P}_{rf}, \tag{22}$$

where $\overline{P}_{rf}$ is the average rf power [see (6)].

If $\overline{R}$ differs from R_1 (the bolometer resistance at initial balance), the resultant error, in fractional form, is

$$\frac{\overline{R} - R_1}{s\overline{P}_{rf}} = 1 - \frac{R_1 - R_0'}{s\overline{P}_{rf}}. \tag{23}$$

$$\text{Now } \Delta R = sP(1 - \epsilon^{-\delta/\tau}) = s\overline{P}_{rf}(1 - \epsilon^{-\delta/\tau})\cdot T/\delta \tag{24}$$

where P is the peak rf power.

Elimination $(R_1 - R_0')$ and $s\overline{P}_{rf}$ in (23) by use of (20) and (24), we have

Fractional error

$$= 1 - \frac{T(1-\epsilon^{-\delta/\tau})}{q\delta}\cdot\frac{(q-\delta)\ln(1+q)+q\delta}{(\delta-q\tau)\ln(1+q)+q(T-\delta)}. \tag{25}$$

For an equal-arm bridge with low galvanometer resistance q is less than $\frac{1}{2}$ for ΔR as high as the burnout limit permits, and is usually less than $\frac{1}{4}$. By making the approximations

$$\ln(1+q) \simeq q - q^2/2 + q^3/3$$
$$1 - \epsilon^{-\delta/\tau} \simeq (\delta/\tau)(1 - \delta/2\tau), \tag{26}$$

(25) may be simplified to

$$\text{Fractional error} = \frac{q[T(1/2 - q/3) - \tau]}{T - q\tau}. \tag{27}$$

The constant r which enters into the determination of q can be written as[10]

$$r = \frac{X[(A+C)D + R_g(A+C+D)] + ACD + AR_g(C+D)}{X[A+C+R_g] + (A+R_g)(C+D) + CD}, \tag{28}$$

where X is the external resistance in series with the bridge, and the various branches are labeled as in Fig. 1. In the case of a constant current source ($X = \infty$) the above expression reduces to

$$r = \frac{(A+C)D + R_g(A+C+D)}{A + C + R_g}. \tag{29}$$

For an infinite impedance detector ($R_g = \infty$), $r = A + C + D$, as given in (15).

Appendix II

Bias Power and RF Impedance Variation Errors

Bias Power Variation Error

The bias power (with rf applied) is ordinarily computed on the assumption that the bolometer resistance is constant with time. The true bias power is the time average of the fluctuating bias power. The bias power variation error may be defined as

$$\text{Per cent error} = \frac{\overline{P}_0 - \overline{P}_0'}{\overline{P}_{rf}} \times 100, \tag{30}$$

where $\overline{P}_0$ is the true time average bias power, $\overline{P}_0'$ the bias power computed on the assumption of constant bolometer resistance, and $\overline{P}_{rf}$ the average rf power. If $I_b(t)$ represents the instantaneous bias current in the bolometer, then

$$\overline{P}_0 = \frac{1}{T}\int_0^T I_b^2(t)R_b(t)dt. \tag{31}$$

The bias current for the case of a constant current source I_0 supplying an equal arm bridge is

$$I_b = I_0\frac{2R(R+R_g)}{R(2R+3R_g) + (2R+R_g)R_b} = \frac{k_1I_0}{1 + k_2R_b}, \tag{32}$$

where R_g is the galvanometer resistance and R the resistance of the equal-bridge arms.

Using (10) to describe the resistance decay of the bolometer between pulses, and assuming negligible pulse width compared to the repetition period T the result of integrating (31) with the aid of (32) is

$$\overline{P}_0 = \left(\frac{k_1I_0}{1 + k_2R_0'}\right)^2\cdot\left[\frac{\Delta R(\tau/T)}{1 + k_2(R_0' + \Delta R)} - R_0(\tau/T)\ln\left(1 + \frac{k_2\Delta R}{1 + k_2R_0'}\right) + R_0'\right]. \tag{33}$$

[10] Equations for the currents in all branches of a bridge circuit can be found conveniently tabulated in Weston Engineering Notes, vol. 2, pp. 6–8; October, 1947. Eq. (28) is easily derived from the equation for the galvanometer current.

The assumed bias power is

$$\overline{P}_0{}' = I_0{}^2 \cdot R/4, \tag{34}$$

and is always greater than $\overline{P}_0$. The error is given by (30).

This error was computed (see Fig. 4) for the case of a PRD 630A bolometer using the following conditions and parameters:

$I_0 = 10^{-2}$ amp $\quad R = 200$ ohms

$\tau = 10^{-4}$ sec $\quad \overline{P}_{rf} = 1.0 \times 10^{-3}$ watts for $\Delta R = 100$ ohms

$T = 10^{-3}$ sec $\quad \overline{P}_0{}' = 5 \times 10^{-3}$ watts.

The average rf power was assumed proportional to the pulse energy and therefore to the resistance ex-

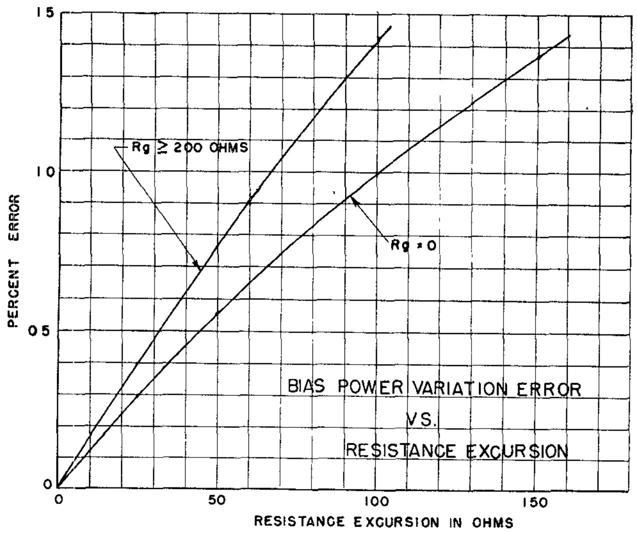

Fig. 4—Error caused by the time variation of the bias power in a bolometer when measuring pulsed rf power. These curves are for a PRD-type 630A bolometer in a 200-ohm bridge fed from a constant current source and were computed for an assumed galvanometer resistance of zero and 200 ohms. The assumed bias power was 5 milliwatts.

cursion ΔR. The case of $R_g = 0$ corresponds very closely to the actual case ($R_g = 10$ ohms), as does the assumption of a constant current supply.

RF Impedance Variation Error

If a matched generator is used, then the error caused by time-dependent mismatch of the bolometer to the rf transmission line is

$$\text{Error} = \frac{\overline{P}_{refl}}{\overline{P}_{rf}} \times 100 \text{ Per cent} \tag{35}$$

where $\overline{P}_{rf}$ is the average incident rf power and $\overline{P}_{refl}$ is the average reflected power.

If the VSWR of the bolometer is known as a function of its dc resistance, $\overline{P}_{refl}$ is readily calculated, since the variation of the dc resistance during a pulse is known. The VSWR ρ was determined experimentally at 3,000 mc and found to depend linearly on R_b as follows:

$\rho = 0.05 + 0.005 R_b$ (for the Sperry 821 barretter),

$\rho = 0.11 + 0.0046 R_b$ (for the *PRD* 630A Bolometer). (36)

Eq. (36) holds for $R_b \geqq 200$ ohms. For $190 \leqq R_b < 200$ ohms, the VSWR was essentially constant and equal to that at 200 ohms.

The per cent error is

$$\frac{100}{\delta} \int_0^\delta \frac{(\rho - 1)^2}{(\rho + 1)^2}\, dt, \tag{37}$$

and can be computed in a straight-forward fashion using (36) and (9). It is plotted in Fig. 5.

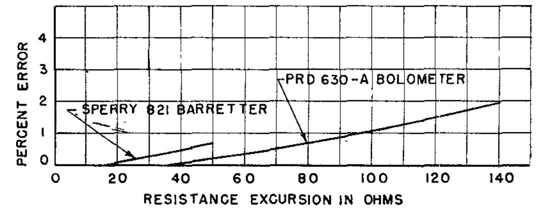

Fig. 5—Error caused by time variation of rf impedance of a bolometer when measuring pulsed power.

Appendix III

Determination of Resistance Excursion Factor (s/τ) by Use of Combined AC and DC Power

If the power input into the bolometer has a component of the form $P \sin \omega t$, where P is the amplitude of the power sinusoid and ω the angular frequency, solution of the heat balance equation (3) gives for the temperature change with respect to ambient,

$$\theta = \frac{\theta_0 \sin (\omega t - \phi)}{\sqrt{1 + (\omega \tau)^2}},$$

$$(\theta_0 \equiv P/\gamma, \quad \tau \equiv c_p/\gamma, \quad \phi \equiv \tan^{-1} \omega \tau). \tag{38}$$

The resistance variation of the bolometer is given by

$$R_b(t) = R_0 + sP \sin (\omega t - \phi)/\sqrt{1 + (\omega \tau)^2}. \tag{39}$$

by application of (4). Eq. (39) shows that the bolometer response to a sinusoidal power input is a sinusoidal resistance variation of the same frequency with phase lag angle ϕ whose value depends on both the applied frequency and the thermal time constant of the bolometer. The amplitude of the resistance variation is likewise dependent on these two parameters and in addition is proportional to the bolometer power sensitivity s.

Now consider the effect of circulating both ac and dc currents through the bolometer in the experimental arrangement of Fig. 6. Let

I_{dc} = dc component of current,

I_{ac} = rms ac component of current,

$R_b(t)$ = instantaneous bolometer resistance $= \overline{R} + r_b(t)$,

$\overline{R}$ = time-average bolometer resistance,

$r_b(t)$ = time-dependent part of bolometer resistance.

The instantaneous power is

$$P(t) = (I_{dc} + \sqrt{2} I_{ac} \sin \omega t)^2 R_b(t). \tag{40}$$

If $r_b(t)$ is small relative to $\overline{R}$, then the applied power

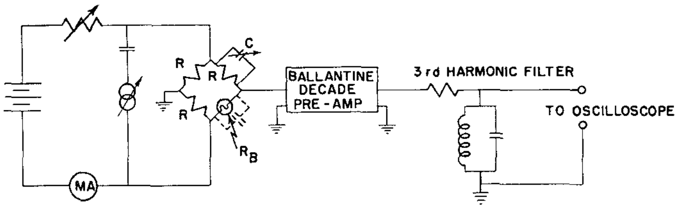

Fig. 6—Experimental arrangement for measuring s/τ by use of a combination of dc and audio power.

$$\Delta R = \frac{\alpha R_0}{c_p} \cdot \text{(pulse energy)}$$

$$\frac{s}{\tau} = \frac{\alpha R_0}{c_p} = \frac{4R^2\omega^2}{7} \cdot \frac{dC}{dP_{dc}} \cdot$$

can be expressed to good approximation as the sum of a steady component

$$P_0 = (I_{dc}^2 + I_{ac}^2)\overline{R} \tag{41}$$

and a time-dependent component

$$p(t) = (2\sqrt{2}I_{dc}I_{ac} \sin \omega t - I_{ac}^2 \cos 2\omega t)\overline{R} \tag{42}$$

By (39)

$$r_b(t) = s\overline{R}\left[\frac{2\sqrt{2}I_{dc}I_{ac} \sin(\omega t - \phi_1)}{\sqrt{(\omega\tau)^2 + 1}} - \frac{I_{ac}^2 \cos(2\omega t - \phi_2)}{\sqrt{(2\omega\tau)^2 + 1}}\right].$$

$$\phi_1 = \tan^{-1} \omega\tau$$

$$\phi_2 = \tan^{-1} 2\omega\tau. \tag{43}$$

Since $\omega\tau \gg 1$ if ω is a sufficiently high audio-frequency,

$$\sqrt{(\omega\tau)^2 + 1} \simeq \omega\tau,$$

$$\sqrt{(2\omega\tau)^2 + 1} \simeq 2\omega\tau,$$

$$\phi_1 \simeq \phi_2 \simeq \pi/2. \tag{44}$$

The voltage $V(t)$ across the bolometer is

$$V(t) = (I_{dc} + \sqrt{2}I_{ac} \sin \omega t)(\overline{R} + r(t)). \tag{45}$$

On carrying through the multiplication and setting aside all dc, second and third harmonic components, the fundamental component $v_f(t)$ is:

$$v_f(t) = \sqrt{2}I_{ac}\overline{R} \sin \omega t$$

$$+ \frac{\sqrt{2}sI_{ac}}{\omega\tau}\left[2P_{dc} + \frac{1}{4} P_{ac}\right] \sin(\omega t - \pi/2), \tag{46}$$

where

$$P_{dc} = I_{dc}^2\overline{R},$$

$$P_{ac} = I_{ac}^2\overline{R}.$$

The first term on the right-hand side of the leading equation in (46) is the in-phase component of voltage that would be generated if the bolometer were purely resistive i.e., not power-sensitive, the second term is a voltage generated by the tendency of the bolometer resistance to follow the applied ac power and lags the applied current by 90 degrees. The total voltage lags the applied current by an angle

$$\psi = \tan^{-1}\left[\frac{s(8P_{dc} + P_{ac})}{4\omega\overline{R}\tau}\right]. \tag{47}$$

If the bolometer were a pure resistance R shunted by a capacitance C, the phase angle of the voltage across its terminals would be

$$\psi = \tan^{-1} \omega CR; \tag{48}$$

accordingly, the effective capacitance generated across the bolometer is

$$C = \frac{s[8P_{dc} + P_{ac}]}{4\omega^2\overline{R}^2\tau} \cdot \tag{49}$$

If the total power is maintained constant but the ratio of dc to ac power varied, C will vary linearly with P_{dc}, and

$$\frac{s}{\tau} = \frac{4\omega^2\overline{R}^2}{7} \cdot \frac{dC}{dP_{dc}} \cdot \tag{50}$$

In practice, the effective capacitance C is balanced out by an equal physical capacitance in an adjacent arm of the bridge. Care must be taken to suppress third harmonic at the null detector by suitable filtering or use of a sharply tuned amplifier.

Acknowledgment

We are indebted to Mr. Leonard Sweet of the Microwave Research Institute who patiently performed many of the measurements and calculations on which this paper is based.[11] Earlier work on this problem was also done by P. Mariotti.[12]

[11] L. Sweet, "A Study of the Error in the Measurement of Pulsed Microwave Power with Bolometers," M.E.E. Thesis, Polytech. Inst. of Brooklyn, Brooklyn, N. Y.; June, 1952.

[12] P. Mariotti, "A Study of Bolometer Errors as a Function of the Pulse Modulation of the RF Signal," M.E.E. Thesis, Polytech. Inst. of Brooklyn, Brooklyn, N. Y.; May, 1948.

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Proceedings or Transactions?

Questions frequently arise—in Editorial Board discussions and among the reviewers of PROCEEDINGS papers—as to what should be published in the PROCEEDINGS OF THE IRE and what should be published in the various TRANSACTIONS. This statement will not settle the matter, but it does clarify the present situation and speculate about future possibilities.

One aspect of the current state of affairs is that of the twenty-three Professional Groups of the Institute, twenty-one publish TRANSACTIONS, while only ten of the twenty-one TRANSACTIONS appear regularly. Thus, in some fields of interest the PROCEEDINGS is the Institute's only facility for publication, and in some it is the only regularly published facility. As long as this situation persists, there must be a certain seeming inconsistency